SADLER MATHEMATICS METHODS UNIT 2

WORKED SOLUTIONS

Chapter 3 Sequences

Exercise 3A

Question 1

$T_3 = 18$

Question 2

$T_5 = 26$

Question 3

$T_3 + T_5$
$= 18 + 26$
$= 44$

Question 4

$T_8 = 38$

Question 5

$3T_2$
$= 3 \times 14$
$= 42$

Question 6

$$
\begin{aligned}
&2T_3 \\
&= 2 \times 18 \\
&= 36
\end{aligned}
$$

Question 7

$$
\begin{aligned}
&3(T_1 + T_2) \\
&= 3(10 + 14) \\
&= 72
\end{aligned}
$$

Question 8

$$
\begin{aligned}
&3(T_1 + T_2 \\
&= 3 \times 10 + 14 \\
&= 44
\end{aligned}
$$

Question 9

$T_9 = 42$

Question 10

$T_{10} = 46$

Question 11

$$
\begin{aligned}
&(T_3)^2 \\
&= 18^2 \\
&= 324
\end{aligned}
$$

Question 12

$(T_2)^3$
$= 14^3$
$= 2744$

Question 13

$T_2 = 8$

Question 14

$T_6 = 20$

Question 15

$T_2 + T_6$
$= 8 + 20$
$= 28$

Question 16

$T_8 = 26$

Question 17

$T_9 = 29$

Question 18

$T_3 + 2T_1$
$= 11 + 2 \times 5$
$= 21$

Question 19

$T_1 + 2T_3$
$= 5 + 2 \times 11$
$= 27$

Question 20

$(T_3 - T_2)^2$
$= (11 - 8)^2$
$= 9$

Question 21

$T_5 = 162$

Question 22

$3T_2$
$= 3 \times 6$
$= 18$

Question 23

$T_1 + T_2 + T_3$
$= 2 + 6 + 18$
$= 26$

Question 24

$T_8 = 1458 \times 3$
$= 4374$

Question 25

$C_3 = 27$

Question 26

$C_6 = 6^3$
$= 216$

Question 27

$C_7 = 343$

Question 28

$C_6 - C_5$
$= 216 - 125$
$= 91$

Question 29

$L_3 = L_1 + L_2$
$= 1 + 3$
$= 4$

Question 30

$L_4 = L_2 + L_3$
$= 3 + 4$
$= 7$

Question 31

$$L_4^{\,2} =$$
$$= 7^2$$
$$= 49$$

Question 32

$$L_5 = 4 + 7 = 11$$
$$L_6 = 7 + 11 = 18$$
$$L_7 = 11 + 18 = 29$$
$$L_8 = 18 + 29 = 47$$
$$2L_8 = 2 \times 47$$
$$= 94$$

Exercise 3B

Question 1

$T_1 = 6, \qquad T_{n+1} = T_n + 4$

Question 2

$T_1 = 28, \qquad T_{n+1} = T_n - 2$

Question 3

$T_1 = 5, \qquad T_{n+1} = T_n + 10$

Question 4

$T_1 = 7.5, \qquad T_{n+1} = T_n + 2.5$

Question 5

$T_1 = 100, \qquad T_{n+1} = T_n - 11$

Question 6

$T_1 = 6, \qquad T_n = T_{n-1} \times 2 = 2T_{n-1}$

Question 7

$T_1 = 0.375, \qquad T_n = 4T_{n-1}$

Question 8

$T_1 = 384, \qquad T_n = 0.25T_{n-1}$

Question 9

$T_1 = 50, \qquad T_n = 3T_{n-1}$

Question 10

$T_1 = 1000, \qquad T_n = 1.1T_{n-1}$

Question 11

Neither

Question 12

Geometric (multiplication by 5 to get next term)

Question 13

Arithmetic (addition of 1.5 to get next term)

Question 14

Arithmetic (subtraction of 11 to get next term)

Question 15

Neither

Question 16

$\frac{160}{128} = 1.25$

$\frac{200}{160} = 1.25$

$\frac{250}{200} = 1.25$ and so on

Geometric

Question 17

Geometric ($3T_n$ indicates multiplication by 3 to get next term)

Question 18

Arithmetic ($T_n + 6$ indicates addition of 6 to get next term)

Question 19

Neither

Question 20

Neither

Question 21

Arithmetic ($T_n - 8$ indicates subtraction of 8 to get next term)

Question 22

Geometric ($0.5T_n$ indicates multiplication by 0.5 to get next term)

Question 23

$T_1 = 8,\ T_2 = 11,\ T_3 = 14,\ T_4 = 17$
$T_{n+1} = T_n + 3$

Question 24

$T_1 = 100,\ T_2 = 97,\ T_3 = 94,\ T_4 = 91$
$T_{n+1} = T_n - 3$

Question 25

$T_1 = 11,\ T_2 = 22,\ T_3 = 44,\ T_4 = 88$
$T_{n+1} = 2T_n$

Question 26

$T_1 = 2048,\ T_2 = 1024,\ T_3 = 512,\ T_4 = 256$
$T_{n+1} = 0.5T_n$

Question 27

a $8400 - 7600 = 7600 - 6800 = 6800 - 600 \ = 800$

Each term is 800 more than previous

b $N_{n+1} = N_n + 800$

Question 28

a Geometric as adding 10% is the same as multiplying by 1.1

b $T_1 = 500$

$T_2 = 500 + 10\% \times 500 = 550$

$T_3 = 550 + 10\% \times 550 = 605$

$T_4 = 605 + 10\% \times 605 = 665.5$

Question 29

a Geometric as adding 25% is the same as multiplying by 1.25

b $T_1 = 1000$

$T_2 = 1000 + 25\% \times 1000 = 1250$

$T_3 = 1250 + 25\% \times 1250 = 1562.5$

$T_4 = 1562.5 + 25\% \times 1562.5 = 1953.125$

Question 30

a Geometric as subtracting 10% is the same as multiplying by 0.9

b $T_1 = 24000$

$T_2 = 24000 - 10\% \times 24000 = 21600$

$T_3 = 21600 - 10\% \times 21600 = 19440$

$T_4 = 19440 - 10\% \times 19440 = 17496$

Question 31

a

n	1	2	3	4	5
T_n	3	8	13	18	23

$T_1 = 3, \qquad T_{n+1} = T_n + 5$

b Arithmetic

Question 32

a

n	1	2	3	4	5
T_n	1.5	3	6	12	24

$T_1 = 1.5, \qquad T_{n+1} = 2T_n$

b Geometric

Question 33

a

n	1	2	3	4	5
T_n	4	9	16	25	36

b Neither

Question 34

a $1296, 1392, 1488, 1584$ after 1, 2, 3, 4 years respectively

b Arithmetic as there is a constant difference of 96

c $T_1 = 1200, \quad T_{n+1} = T_n + 96$

Question 35

$T_1 = 4,\ T_2 = 5,\ T_3 = 6,\ T_4 = 7$

$T_1 = 4 \qquad T_{n+1} = T_n + 1$

Question 36

$T_1 = 45\,000,\ T_2 = 46\,500,\ T_3 = 48\,000$

$T_1 = 45\,000 \qquad T_{n+1} = T_n + 1500$

Question 37

$T_1 = 68\,000$ (2014)

$T_2 = 68\,000 \times 1.05 = 71\,400$ (2015)

$T_3 = 71\,400 \times 1.05 = 74\,970$ (2016)

$T_4 = 74\,970 \times 1.05 = 78\,718.50$ (2017)

$T_1 = 68\,000 \qquad T_{n+1} = 1.05T_n$

Question 38

$T_1 = 1500$ (2014)

$T_2 = 1500 \times 1.08 = 1620$ (2015)

$T_3 = 1620 \times 1.08 = 1749.6$ (2016)

$T_1 = 1500 \qquad T_{n+1} = 1.08T_n$

Question 39

$T_1 = 36\ 000$

$T_2 = 36\ 000 \times 0.85 = 30\ 600$

$T_3 = 30\ 600 \times 0.85 = 26\ 010$

$T_1 = 36\ 000 \qquad T_{n+1} = 0.85T_n$

Exercise 3C

Question 1

$a = 11, d = 5$

$T_{100} = a + 99d$

$= 11 + 99 \times 5$

$= 506$

Question 2

$a = -8, d = 3$

$T_{100} = a + 99d$

$= -8 + 99 \times 3$

$= 289$

Question 3

$T_1 = a = 23, d = 8$

$T_{100} = a + 99d$

$= 23 + 99 \times 8$

$= 815$

Question 4

$T_1 = a = 78, d = -2$

$T_{100} = a + 99d$

$= 78 + 99 \times (-2)$

$= -120$

Question 5

$T_1 = a = 5, r = 2$

$T_{25} = ar^{24}$

$= 5 \times 2^{24}$

Question 6

$T_1 = a = 1.5, r = 4$

$T_{25} = ar^{24}$

$= 1.5 \times 4^{24}$

Question 7

$T_1 = a = 8, r = 3$

$T_{25} = ar^{24}$

$= 8 \times 3^{24}$

Question 8

$T_1 = a = 11, r = 2$

$T_{25} = ar^{24}$

$= 11 \times 2^{24}$

Question 9

$T_8 = 223$

Question 10

$T_{20} = 3\,495\,265$

Question 11

$T_{19} = 774\,840\,977$

Question 12

$T_{45} = 6$

Question 13

Day	1	2	3	4	5
Items	48	51	54	57	60

$T_n = 48,$ $T_{n+1} = T_n + 3$

By classpad or

$T_{15} = a + 14d$

$= 48 + 14 \times 3$

$= 90$

She completes 90 items of day 15.

Question 14

$T_n = a + (n-1)d$

$= a + dn - d$

$= dn + (a - d)$

T_n produces a sequence which increases by d for each unit increase in the n-value, which is by definition, the gradient.

When $n = 0, T_n = a - d \Rightarrow$ co-ordinates are$(0, a - d)$

Question 15

$T_n = ar^{n-1}$

Exponential relationships are of the form $y = ka^x$ which is similar to our T_n formula.

$$\begin{aligned} T_n &= ar^n \\ &= a \times r^n \times r^{-1} \\ &= a \times r^n \times \frac{1}{r} \\ &= \frac{a}{r} \times r^n \end{aligned}$$

When $n = 0, T_n = \frac{a}{r}$ co-ordinates are $(0, \frac{a}{r})$

Question 16

As $n \to \infty$, the nd term in the expression $a + (n-1)d$ will dominate. Thus as $n \to \infty$, T_n will become increasingly large and positive if $d > 0$ and increasingly large and negative if $d < 0$.

Question 17

As $n \to \infty$, the n in the expression ar^{n-1} will dominate. Thus as $n \to \infty$, if $r > 1, .T_n$ will become increasingly large, either positively or negatively dependent on the sign of the constant *a*.

If $r < -1$, T_n will become increasingly large, alternating between large negative and large positive.

If $-1 < r < 1$, T_n will become smaller and smaller, maintaining the sign of the constant *a* if r is positive and alternating between small positive and small negative if r is negative.

Question 18

$a = 8, d = 3$

$T_1 = 8,\ T_2 = 11,\ T_3 = 14,\ T_4 = 17$

$T_{50} = a + 49d$

$= 8 + 49 \times 3$

$= 155$

$T_{100} = a + 99d$

$= 8 + 99 \times 3$

$= 305$

Question 19

$a = 100, d = -3$

$T_1 = 100,\ T_2 = 97,\ T_3 = 94,\ T_4 = 91$

$T_{50} = a + 49d$

$= 100 + 49 \times (-3)$

$= -47$

$T_{100} = a + 99d$

$= 100 + 99 \times (-3)$

$= -197$

Question 20

$a = 11, r = 2$

$T_1 = 11,\ T_2 = 22,\ T_3 = 44,\ T_4 = 88$

$T_{15} = ar^{14}$

$= 11 \times 2^{14}$

$= 180\ 224$

$T_{25} = ar^{24}$

$= 11 \times 2^{24}$

$= 184\ 549\ 376$

Question 21

$a = 2048,\ r = 0.5$

$T_1 = 2048,\ T_2 = 1024,\ T_3 = 512,\ T_4 = 256$

$T_{16} = ar^{15}$

$= 2048 \times (0.5)^{15}$

$= 0.0625$

Question 22

a $T_n = a + (n-1)d$

$= 9 + 6(n-1)$

$= 9 + 6n - 6$

$= 6n + 3$

b $T_n = a + (n-1)d$

$= 7 + 1.5(n-1)$

$= 7 + 1.5n - 1.5$

$= 1.5n + 5.5$

Question 23

a $T_n = ar^{n-1}$

$= 3 \times 2^{n-1}$

b $T_n = ar^{n-1}$

$= 100 \times (1.1)^{n-1}$

Question 24

a $a = 2, d = 7$

$T_{123} = a + 122d$

$= 2 + 122 \times 7$

$= 856$

b $T_{500} = a + 499d$

$= 2 + 499 \times 7$

$= 3495$

c By classpad, solve $1\,000\,000 < 2 + 7(n-1)$

or

$999\,998 < 7(n-1)$

$142\,856\frac{6}{7} < n - 1$

$142\,857\frac{6}{7} < n$

The 142 858th term

Question 25

a $a = 0.0026, r = \dfrac{0.013}{0.0026} = 5$

$T_{12} = ar^{11}$

$= 0.0026 \times 5^{11}$

$= 126\,953.125$

b By classpad, solve $0.0026 \times 5^{n-1} > 1\,000\,000$

$n - 1 > 12.28$

$n > 13.28$

The 14th term

Question 26

a $a = 20\,000\,000,\ r = \dfrac{15}{20} = 0.75$

$T_{12} = ar^{11}$

$= 20\,000\,000 \times 0.75^{11}$

$= 844\,700$ (to nearest 100)

b By classpad, solve $20\,000\,000 \times (0.75)^{n-1} < 1$

$n - 1 > 58.4$

$n > 59.4$

The 60th term

Question 27

$T_n = n^3$

$T_1 = 1,\ T_2 = 8,\ T_3 = 27,\ T_4 = 64$

Neither

Question 28

$T_{19} = a + 18d = 61$

$T_{41} = a + 40d = 127$

$T_{41} - T_{19} = 22d = 66$

$d = 3$

a $T_{20} = T_{19} + 3$

$= 61 + 3$

$= 64$

b $a + 18 \times 3 = 61$

$a + 54 = 61$

$a = 7$

Question 29

$T_{50} = a + 49d = 1853$

$T_{70} = a + 69d = 1793$

$T_{70} - T_{50} = -20d = 60$

$d = -3$

a $T_{51} = T_{50} - 3$

$= 1853 - 3$

$= 1850$

b $a + 49 \times (-3) = 1853$

$a - 147 = 1853$

$a = 2000$

Question 30

$T_{10} = ar^9 = 98\,415$

$T_{13} = ar^{12} = 2\,657\,205$

$\frac{T_{13}}{T_{10}} = \frac{2\,657\,205}{98\,415}$

$\frac{ar^{12}}{ar^9} = 27$

$r^3 = 27$

$r = 3$

a $T_{14} = 2\,657\,205 \times 3$

$= 7\,971\,615$

b $T_{10} = a(3)^9 = 98\,415$

$a = \frac{98\,415}{3^9}$

$= 5$

Question 31

$T_7 = ar^6 = 28\,672$

$T_9 = ar^8 = 458\,752$

$$\frac{T_9}{T_7} = \frac{458\,752}{28\,672}$$

$$\frac{ar^8}{ar^6} = 16$$

$$r^2 = 16$$

$$r = \pm 4$$

Given $r < 0, r = -4$

a $T_{10} = 458\,752 \times (-4)$

$= -1\,835\,008$

b $T_{10} = a(-4)^6 = 28\,672$

$$a = \frac{28\,672}{(-4)^6}$$

$= 7$

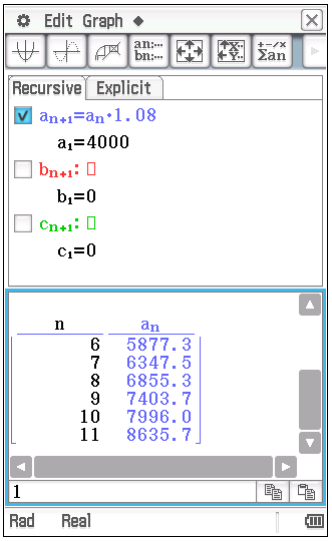
Question 32

$A = 4000 \times 1.08^t$ t years

$A = 4000 \times 1.08^{10}$

$= \$8635.70$

Recursive formula: $T_1 = 4000, \quad T_{n+1} = 1.08T_n$



Question 33

$V = 600\,000 \times 1.056^t \quad t$ years

$600\,000 \times 1.056^t = 2\,000\,000$

By classpad, $t = 22.1$

In the 23rd year.

Recursive formula: $T_1 = 600\,000, \quad T_{n+1} = 1.056T_n$

Question 34

$T_1 = 4520$

$T_{n+1} = 1.08T_n + 200$

$T_{10} = \$11\,533.01$

Question 35

$T_1 = 4120$

$T_{n+1} = 1.08T_n - 200$

$T_{10} = \$5738.39$

Question 1

a Quadratic

b Exponential

c Linear

d Quadratic

e Reciprocal

f Linear

g Linear

h Quadratic

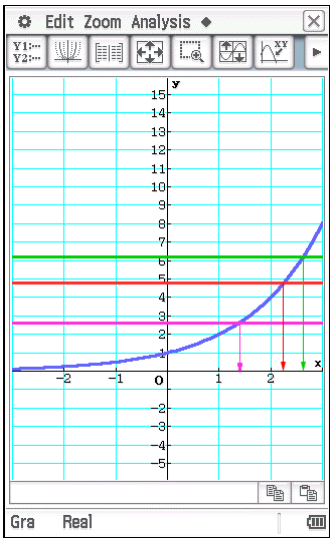
i Quadratic

j Reciprocal

k Linear

l Exponential

Question 2



a $x \approx 2.3$

b $x \approx 2.6$

c $x \approx 1.4$

Question 3

a $8 = 2^3$

$n = 3$

b $\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$

$n = -3$

c $\frac{1}{2} = 2^{-1}$

$n = -1$

d $\sqrt{2} = 2^{0.5}$

$n = 0.5$

e $1 = 2^0$

$n = 0$

f $\sqrt{8} = (2^3)^{0.5} = 2^{1.5}$

$n = 1.5$

g $\frac{1}{64} = \frac{1}{2^6} = 2^{-6}$

$n = -6$

h $2\sqrt{2} = 2^1 \times 2^{0.5} = 2^{1.5}$

$n = 1.5$

Question 4

$T_2 = ar = 6$

$T_5 = ar^4 = 20.25$

$$\frac{T_5}{T_2} = \frac{20.25}{6}$$

$$\frac{ar^4}{ar} = 3.375$$

$$r^3 = 3.375$$

$$r = 1.5$$

$T_2 = a(1.5) = 6$

$$a = \frac{6}{1.5}$$

$$= 4$$

a $T_n = ar^{n-1}$

$= 4(1.5)^{n-1}$

b

$T_n = 4(1.5)^{n-1}$

$= 4 \times (1.5)^n \times (1.5)^{-1}$

$= 4 \times (1.5)^n \times \left(\frac{3}{2}\right)^{-1}$

$= 4 \times \frac{2}{3} \times 1.5^n$

$= \frac{8}{3} \times 1.5^n$

Question 5

$a = 1, \quad r = \sqrt{3}$

$T_{11} = ar^{10}$

$= 1 \times (\sqrt{3})^{10}$

$= 243$

Question 6

a $4^x = 64$

$= 4^3$

$x = 3$

b $4^x = \frac{1}{64}$

$= \frac{1}{4^3}$

$= 4^{-3}$

$x = -3$

c $4^x = \frac{1}{4}$

$= 4^{-1}$

$x = -1$

d $64^{0.5} = x$

$\sqrt{64} = x$

$x = 8$

e $x^2 = 64$

$x = \pm 8$

f $4^8 = 4^x \times 4^{-3}$

$= 4^{x-3}$

$8 = x - 3$

$x = 11$

Question 7

a $16^{0.5}$

$= \sqrt{16}$

$= 4$

b $16^{\frac{3}{2}}$

$= (2^4)^{\frac{3}{2}}$

$= 2^6$

$= 64$

c $27^{\frac{2}{3}}$

$= (3^3)^{\frac{2}{3}}$

$= 3^2$

$= 9$

d $25^{-0.5}$

$= (5^2)^{-0.5}$

$= 5^{-1}$

$= \frac{1}{5}$

e $\left(\frac{1}{4}\right)^{-0.5}$

$= (2^{-2})^{-0.5}$

$= 2^1$

$= 2$

Question 8

a $T_{1+1} = (-1)^1 T_1$

$= -1 \times 4$

$= -4$

b $T_3 = T_{2+1}$

$= (-1)^2 T_2$

$= 1 \times (-4)$

$= -4$

$T_4 = T_{3+1}$

$= (-1)^3 T_3$

$= (-1) \times (-4)$

$= 4$

$T_5 = T_{4+1}$

$= (-1)^4 T_4$

$= 1 \times 4$

$= 4$

c Neither as 4, –4, –4, 4, 4 has no common ratio or difference

Question 9

a $T_{1+1} = (-1)^1 2T_1$

$= -1 \times 2 \times 1$

$= -2$

b $T_3 = T_{2+1}$

$= (-1)^2 2T_2$

$= 1 \times 2 \times (-2)$

$= -4$

$T_4 = T_{3+1}$

$= (-1)^3 2T_3$

$= (-1) \times 2 \times (-4)$

$= 8$

$T_5 = T_{4+1}$

$= (-1)^4 2T_4$

$= 1 \times 2 \times 8$

$= 16$

c Neither as 1, –2, –4, 8, 16 has no common ratio or difference

Question 10

a $a = 5k + 3,\ d = 5 - k$

$T_{10} = a + 9d$

$= (5k + 3) + 9(5 - k)$

$= 5k + 3 + 45 - 9k$

$= 48 - 4k$

b $T_{20} = a + 19d$

$= (5k + 3) + 19(5 - k)$

$= 5k + 3 + 95 - 19k$

$= 98 - 14k$

$98 - 14k = 91$

$14k = 7$

$k = 0.5$

$T_{21} = 91 + (5 - 0.5)$

$= 95.5$

Question 11

$a^{4+3} = a^7$

Question 12

$12x^{2+1}y^{1+3} = 12x^3y^4$

Question 13

$\frac{15a^3b}{10ab^3} = \frac{3a^2}{2b^2}$

Question 14

$9a^2 \times 8a^6b^3$

$= 72a^8b^3$

Question 15

$$\frac{9a^2}{8a^6b^3} = \frac{9}{8a^4b^3}$$

Question 16

$$\frac{6 \times 8b}{a} = \frac{48b}{a}$$

Question 17

$$\frac{2a^2a^3}{b\,b^4} = \frac{2a^5}{b^5}$$

Question 18

$$\frac{k^3(k^4+1)}{k^3} = k^4 + 1$$

Question 19

$$\frac{p^5(1-p^3)}{p^2} = p^3(1-p^3)$$

Question 20

$$\frac{5^k \times 5^2}{5^k \times 5^{-1}}$$
$$= 5^{2\times} \times 5^1$$
$$= 5^3$$
$$= 125$$

Question 21

$$\frac{5^n \times 5^2 - 50}{5^n - 2}$$
$$= \frac{25(5^n - 2)}{5^n - 2}$$
$$= 25$$

Question 22

$$\frac{2^n \times 2^3 + 8}{3 \times 2^n + 3}$$
$$= \frac{8(2^n + 1)}{3(2^n + 1)}$$
$$= \frac{8}{3}$$